The basic cosmological formula in model of the Static Universe

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Abstract
In article the deduction of the Basic cosmological formula is demonstrated on a minimum quantity of requirements in model of the Static Universe, the estimate of value of the usual mechanism of a dispersion of a velocity of light in intergalactic plasma is made, an appearance of representation about the sped up expanding of space in the models viewed by S.Perlmutter is explained.

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1. Introduction
Models of the Static Universe guess static properties of the perpetual space of the Universe in which a transformation of kinds of a matter is perpetual in a time. The author of a theory of relativity (RT) A. Einstein in 1917 viewed a model of the Static Universe, perpetual in a time, but terminating in space. [1].

For models of the Static Universe the explanation of formation of a cosmological redshift (CR) because of different rate of a clock of a source and the receiver does not suit, as it happens in models with the dilating Universe, deduced by Fridman and Gamov from the Basic equation of TR and nonsteady metric.

Therefore A.A.Belopolsky was the first who has offered a hypothesis according to which light quants lose energy in process of their spatial motion and that was reflected in decrease of their frequency [2]. In 1930 A.A.Belopolsky has published the second article [3] in which has prolonged an arguing of hypothesis of "tired quanta" and has made its estimate on available observations.

Irrespective of Belopolsky, in 1929 F.Zwicky has offered the Concept of tired light [4] according to which photons lose energy in collisions with other particles of interstellar medium. Model of the Static Universe and/or the Concept of tired light in the Static Universe had been viewed also by F.Hoyle [5], H.Alfven [6], A.Assis [7], E.Lerner [8], V.Troitsky [9] and many other known scientists.

The basic cosmological formula in model of the Static Universe is:

\[ R = R_0 \ln(z+1), \] (1)
Its equivalent view:

\[ v = v_0 \cdot \exp(-R/R_0), \]  

where \( R \) - distance between a source and receiver when CR is equal \( z \); \( R_0 \) - distance between a source and receiver when \( z = e^{-1} \); \( v_0 \) - frequency rayed (on clock of a source); \( v \) - frequency received (on clock of the receiver): \( z+1=v_0/v \).

Formulas (1) and/or (2) were derived by several authors from various sets of requirements, in particular: V.Nernst [10, formula (1)], N.Zhuk [11]. N. Wright even has derived this formula for model of the expanding Universe, however at this deduction he has not considered a time dilatation which does not allow to exist in the specified model a relation of kind \( D_{\text{now}} = (c/H_0) \ln (1+z) \). [12]

Propagation of light from distant galaxies happens mainly in intergalactic space which is filled by intergalactic medium. Properties of this medium are close to properties of terrain clearance vacuum, however differ from them a little. Properties of medium determine: a velocity with which the light quantum of frequency \( \nu \) is spreading (a velocity dispersion); time \( T \) of a transiting of a light quantum of a distance \( R \); a percent of light quantums from a source which will not get to the receiver because of uptake and the scattering; a part of a light quantum energy which remains in medium at transiting by it of a distance \( R \) (tired light).

In this article the minimum set of requirements is offered, which sufficient for a deduction of exponential dependence of cosmological redshift \( z \) of received light, and distance \( R \) from a source of this light.

2. Minimum requirements for a deduction of the basic cosmological formula

For a deduction of the basic cosmological formula we will be based on the basic cosmological statements:

A. The space of the Universe has Static properties of the perpetual space in which a transformation of kinds of a matter is perpetual in a time.

B. Independence of the cosmological redshift \( z \) from a direction on a source and from viewed frequency \( \nu(0) \) light quantums radiated by a source:

\[ z(R) = \nu(0)/\nu(R) - 1, \]  

where \( \nu(0) \) is the frequency of a ray of light (measured on a source), \( \nu(R) \) is the frequency of the same ray of light (measured by the receiver, being on a distance \( R \) from a source).

Statement A. is a postulate. Statement B. is a postulate, too, and, together with it, the observational fact for frequencies, accessible to measuring, if to subtract, mainly, a gravitation effect and a Doppler effect which arise at peculiar moving of the Earth and far sources of light in an outer space.

3. Deduction of the basic cosmological formula

Let in static space some viewed area of space is filled by a homogeneous environment. We consider a source and the receiver as immobile in space.

Let's consider a ray of light spreading along a straight line, for example, of an axis of co-ordinates and gradually losing energy of light quantums. We designate \( \nu_x(y) \) - the frequency of light quantums measured by the receiver, being on a distance \( y \) from a source disposed in a point \( x \), and raying frequency \( \nu_x(0) \). The Light quantum radiated in a point «0» with frequency \( \nu_0(0) \), will have frequency \( \nu_0(x) \) in a point \( x \), and in a point \( (x+y) \) - frequency \( \nu_0(x+y) \). For this ray in the point \( x \) a coming frequency \( \nu_0(x) \) is equal to an “emitted” frequency \( \nu_x(0) \) in the same point \( x \), so we must write: \( \nu_0(x) = \nu_x(0) \). But the ray of light which has come to a point \( (x+y) \) with frequency \( \nu_0(x+y) \), transited through a point \( x \), i.e., this ray can be viewed as radiated in a point \( x \) with frequency \( \nu_x(0) \). And by the designation given above, its frequency on distance \( y \) is equal to \( \nu_x(y) \), hence, \( \nu_0(x+y) = \)
Let's designate $\Phi(y)$ a value $\Phi(y) = v_s(y)/v_s(0)$. Owing to Statement B, value $v_s(y)/v_s(0) = z(y) + 1$ does not depend on frequencies, and owing to homogeneity of medium - does not depend on an index point $x$, and depends only on distance $y$.

As within the limits of the given requirements for the ratio of frequencies the next equalities are fulfilled:

$$v_0(y+x)/v_0(0) = v_0(x+y)/v_0(0) = [v_0(x+y)/v_0(x)] [v_0(x)/v_0(0)] = [v_s(y)/v_s(0)] [v_0(x)/v_0(0)],$$

so for function $\Phi()$ by means of these equalities we obtain the equation

$$\Phi(y+x) = \Phi(y) \cdot \Phi(x).$$

The solution of this equation is the exponential curve

$$\Phi(R) = \exp(R/R_0) = v_0(R)/v_0(0) = z(R)+1,$$

where $R_0$ is a constant (characterising medium), it is not dependent on distance $R$, frequency and a start point. The value $R_0$ we will term as Hubble's Distance, it is equal to the distance transited by a light quantum by the time when its frequency will decrease in $(e-1)$ time.

Thus, exponential dependence of CR $z$ (in received light) from distances $R$ to a source of this light is proved.

$$z(R) = \exp(R/R_0)-1.$$

In particular, this relation should be fulfilled and for $R/R_0 << 1$. That allows to calculate $R_0$ as it is known that CR $z$ for the close source with $R << R_0$ is directly proportional to distance $R$ up to it (Hubble's law):)

$$z = RH/c,$$

where $c$ is velocity of extending of light quantum in vacuo, $H$ - Hubble constant.

In model of the Static Universe a value $H$ has dimensionality of frequency and characterises rate of an energy loss of quanta of spreading light. However in a reality light is spreading not in terrain clearance vacuum, but in dilute medium, light velocity is less $c$, besides, this velocity depends on frequency, and, possibly, a value $H$ depends on frequency, too, but we cannot note it as measurement accuracy of $H$ does not exceed 0.001. Also we will notice that all areas of space filled with medium with equal properties, will possess equal value $H$.

As we have $\exp(R/R_0)-1 \approx R/R_0$ for $R/R_0 << 1$ so we obtain that Hubble distance $R_0$ is equal $c/H$:

$$R_0 = c/H.$$  \hspace{1cm} (7)

And the basic cosmological formula of model of the Static Universe looks like.

$$z(R) = \exp(RH/c)-1.$$  \hspace{1cm} (8)

This relation operates and for large distances, because in model of the Static Universe the value $H$ is identical on any section of a trajectory of a light ray in homogeneous (intergalactic) medium.

Thus, the deduction of the basic cosmological formula of model of the Static Universe is obtained only from the unique theoretical guess about static space of the Universe, with use of two observant facts. It is impossible to make less guesses in this model.

Let's notice that, according to formulas (1), (3) and (5), dependence of received frequency $v(R)$ from emitted frequency $v_0$ looks like:

$$v(R) = v_0 \cdot \exp(-R/R_0).$$  \hspace{1cm} (9)

As the value $R_0$ characterises the medium so for various mediums $R_0$ it can be differed, in particular, for interstellar, nearby-galactic and intergalactic mediums. Possibly, for example, that the medium around quasars has essentially smaller the value $R_0$ that in an modern model, at adoption of usual value of a Hubble constant, leads to the demand of essentially larger luminosity of quasars in comparison with luminosity of usual galaxies.

As we view not the vacuum but medium, so in the formula (7) of relations of medium's characteristics there is a group velocity $c(\nu)$ of light extending in this medium but not a value «$c$» -
velocity of extending of light in vacuo. As the intergalactic medium represents almost terrain clearance vacuum, so it has very close to 1 a refractive index \( n(\nu) \) \[ 13 \], and the value \( c(\nu) = c/n(\nu) \) should be very close to \( \ll c \) in an observable frequency band. But characteristics \( R_0 \) does not depend on frequency, as \( z \) and \( R \) do not depend on frequency (see formulas (1) and (3), and a deduction f.(3) does not depend on the value \( H \)). Therefore, to compensate a modification \( c(\nu) \) in the formula (7), characteristics \( H(\nu) \) even a homogeneous environment should depend too from \( \nu \), that is,

\[
R_0 = c(\nu)/H(\nu) = c/(n(\nu)\cdot H(\nu)) \tag{10}
\]

\[
H(\nu) = c/(n(\nu)\cdot R_0) \tag{11}
\]

\[
z(R) = \exp[R\cdot H(\nu)/c(\nu)] - 1. \tag{12}
\]

For an inhomogeneous medium it is necessary to divide the path transited by a light quantum, into small sections \( \Delta R_i \) with a homogeneous environment on them (on such a different section there can be a different characteristics \( R_{0i} \) of medium), and for evaluation of resultant frequency \( \nu(R) \) of the viewed quantum it is necessary to multiply together values \( \exp(\Delta R/R_{0i}) \), according to the formula (2) that in a limit matches the formula.

\[
z(R) = \exp \left[ \int_0^R \frac{\delta r}{R_0(r)} \right] - 1. \tag{13}
\]

Thus, here in model of the Static Universe for each point of space, depending on a state in it of medium, correlation of two characteristics of medium was considered: Hubble distance \( R_0 \) and frequency spectrum \( H(\nu) \).

4. The Estimate of dependence of a light velocity from frequency

At the present stage of researches for \( H(\nu) \) is supposed a constant value (not dependent on frequency \( \nu \)) as the relative precision of its measuring does not exceed 0.001, but demanded accuracy better than \( 10^{-12} \). In it is the reason, that the value \( V(\nu) \) was exchanged by the value \( \ll c \) (closely to it) and in the formula (8) and in Hubble's law (6). However viewing of the reasons of a stretching of light curves for SN Ia showed, that for the value \( V(\nu) \) there are strict enough restrictions from above and from below, that is, it is impossible to skip a change of it.

It is obvious that in intergalactic medium a velocity of extending of light depends on frequency. The analysis of results of researches that had been made by S. Perlmutter [14], P. Newgent [15] and G. Goldhaber [16] unambiguously showed, that such dependence exists. So, for far Supernovas all of them have discovered the relative shift of a general maximum of luminosity SN Ia into earlier epoch, and also are discovered shifts into different epochs of the maximums of luminosity SN Ia in each observable frequency band into different epochs, that is obliged to happen at a dispersion of velocity of extending of light. And, at last, in these results there is the relative shifts of spectrums of different frequencies into different epochs that is obliged to be in case of a dispersion of a velocity of light too (in model of the Static Universe), but cannot be explained by modification of rate of a time in any way (in model with expansion of space) as at a modification of rate of a time the spectrums should be stretched, but not be intermixed at shift into different epochs.

4.1. The Estimate of a dispersion of a light velocity on an electron concentration in plasma

Now we shall try to apply the mechanism of an estimate of refractive index on an electron concentration in plasma to an estimate of a velocity of light in intergalactic medium. In §4.7 K.A.Postnov's textbook the formula of refractive index in space plasma and the formula of a delay time of signals \[ 17, \text{s.117-118} \] are given:

«Refractive index for radio waves with frequency \( \omega \) with an electron concentration \( n_e \) is equal in plasma

\[
n = \sqrt{1 - \omega_p^2/\omega^2} < 1, \tag{4.16}
\]
where a natural resonant frequency of electrons of plasma (Langmuir frequency) in the field of ions

$$\omega_p = \sqrt{4\pi e^2 n_e/m_e} \approx 5.64 \cdot 10^4 \sqrt{n_e}, (4.17)$$

The phase velocity of extending of an electromagnetic wave with frequency $\omega$ is $v_\phi = c/n$ ($c$ is a velocity of light), and a group velocity is $v_g = cn$. Emanation of pulsars is polychromatic, hence, on different frequencies a time of arrival of impulses from distance $l$ will be various:

$$t = l/v_g = l/(cn) \approx (l/c)(1 + \frac{1}{2}(\omega_p/\omega)^2),$$

whence a delay time of a low-frequency signal in a homogeneous environment

$$\Delta t(\omega) = \frac{1}{2}(l/c)\left(\frac{\omega_p}{\omega}\right)^2 = \frac{2\pi e^2}{m_e c}(\frac{n_e}{\omega^2}), (4.18)$$

i.e. at the given value $\omega$ it is proportional to the value of a measure of dispersion - to an integral from an electron concentration along a light beam:

$$DM = \int n_e dl, (4.19)$$

Usually for pulsars $10 < DM < 500$ pk/sm$^3$. Generally

$$\Delta t_{1,2} = \int \left(\frac{dt}{v_\phi(\omega_1)} - \frac{dt}{v_\phi(\omega_2)}\right) \approx 4.6[m/c](\lambda_1^2 - \lambda_2^2) \times DM, (4.20)$$

where the wave length $\lambda$ is expressed in sm. Averaged along a beam of light the density of an electronic component of interstellar gas strongly depends on a direction in the Galaxy. Its mean in a plane of the Galaxy is about 0.03 particles in 1 sm$^3$.

Let's note that the right member of the formula (4.20) is designed for enough short distance as in it is not considered the modification of frequency of light in process of growth of the distance transited by it.

The estimate of the value of Langmuir frequency for intergalactic space where the electron concentration is $n_e=1.95\cdot10^{-4}$ sm$^{-3}$ [18], will give $\omega_p=790$ Hz, $v_p = \omega_p/(2\pi) = 125$ Hz. It matches to a wave length more than 2000 km. Existing devices fix waves of essentially smaller length, for them the formula (4.16 [17] is applicable. But we gain refractive index of intergalactic medium $n<1$, that is, in K.A.Postnov's textbook [17] definition is using, on which refractive index is inverse to the value given in its definition in «Physical encyclopaedia» [13],

Let's pay attention that for frequencies of a visible band with $\nu$ the order of $10^{14}$ Hz a velocity of extending of light is on the value, smaller than $c\cdot10^{-10}$ - the accepted accuracy of a velocity of light, therefore we have no possibility to fix such modification of a velocity of light:

$$c - cn = c \cdot \left(1 - \sqrt{1 - \frac{\omega_p^2}{\omega^2}}\right) \approx c \frac{\omega_p^2}{2\omega^2} \approx c \cdot 10^{-25} \ [sm/s], (14)$$

It a very small modification of the light velocity, not matching to an observable extension of duration of explosions SN Ia, that is, or the electron concentration should be more, or such mechanism can give the contribution to light velocity's dispersion, but it is not the main and then there should be other properties of intergalactic medium, which decelerate light.

4.2. The Estimate of a dispersion of a light velocity on magnification of duration of a light curve

It is known that the basic contribution to an energy flow received by us from SN Ia, is in a band of the visible light rayed at such explosion [14]. In the same band (from 4.0-1014 Hz to 7.5-1014 Hz) the most of galaxies radiates the basic energy [19]. We shall try to estimate a required difference of medial velocities of light for this frequency band, rayed on distance $R$. It is known that in this case we observe the magnification of duration of a light curve (in comparison with duration of nearby SN Ia) approximately in $(z+1)$ time, and a decrease of received frequencies in $(z+1)$ time. We shall try to estimate a demanded concentration of free electrons in intergalactic plasma on a
known extension of a light curve.

Time $T(R)$ of transiting of distance $R$ by a light quantum with initial frequency $\nu_0$ will be

$$T(R) = \int_0^R \frac{dr}{c(\nu)} = \frac{1}{c} \int_0^R n(\nu)dr.$$  \hspace{1cm} (15)

Let’s present refractive index $n(\nu)$ as $n(\nu) = 1 + \frac{A}{\nu^2} = 1 + \frac{A}{2\nu_0^2 \exp(-2r/R_0)}$, where $A$=const. Then

$$T(R) = \frac{R}{c} + \frac{A R_0}{2c \nu_0^2} \left( \frac{e^{2R/R_0} - 1}{\nu_0^2} \right) = \frac{R}{c} + \frac{A}{2\nu_0^2} \left( \frac{z^2 + 2z}{1} \right).$$  \hspace{1cm} (16)

And the difference of a time of arrival of two simultaneously radiated frequencies $\nu_1$ and $\nu_2$ will be

$$\Delta T(R) = \frac{A}{\nu_0^2} \left( \frac{z + z^2/2}{1} \left( \frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) \right).$$  \hspace{1cm} (17)

This value matches a difference of a time of arrival of maximums of luminosity of SN Ia in bands with medial frequencies $\nu_1$ and $\nu_2$.

In practice [13-15] we have for nearby SN Ia the simultaneously received (and rayed) maximums of all bands, and medial duration $T_0$ of explosion about 6 weeks, that is, $T_0 = 3.63 \cdot 10^7$ seconds. For far SN Ia (but with $z < 1$) we have maximums of all bands received not simultaneously and medial duration of explosion $(1+z)T_0$, which is an emitted duration of explosion $T_0$ plus a delay of frequencies, mainly, of a visible band, $\Delta T(R)$. Hence, $T_0 + \Delta T(R) \approx (1+z)T_0$ and $\Delta T(R) \approx zT_0$, whence for given $T_0$, $\nu_1$ and $\nu_2$, and also for $H_0=73 \pm 2$ km/s/Mpk [20], we shall compute the demanded value of constant $A$:

$$A \approx T_0H_0 \cdot \frac{\nu_2^2 - \nu_1^2}{\nu_2^2 - \nu_1^2} = \frac{3.63 \cdot 10^7 \cdot 73}{3.08 \cdot 10^{10}} \cdot 22.36 \cdot 10^{28} = 1.92 \cdot 10^{18} \left[ \text{GHz} \right].$$  \hspace{1cm} (18)

In terms of the Postnov’s textbook we get demanded values of Langmuir frequency $\nu_p=1.39$ GHz and electron concentrations $n_e=6.0 \cdot 10^8$ particles in 1 sm$^3$. Thus velocities of light waves of a visible band will be in the necessary interval from $c(1 - 13 \cdot 10^{-12})$ to $c(1 - 3.4 \cdot 10^{-12})$.

As value of a demanded electron concentration was gained on many orders more than its estimate in a plane o our Galaxy [17], then the given mechanism considering only an electron concentration in plasma, does not match necessary value of a dispersion of a light velocity, and is required other mechanism considering not only this property of electrons in medium, for example, a normal distribution of velocities of light photons (attempt of numerical estimate of it was made in G.Golushko's article [21]).

5. Increase of distances from the past toward the future

According to the basic cosmological formula of model of the static Universe we shall view increase of distances (in units $R_0$) from the past to the future, these distances match the values $z$ from 11 to 0 with a step 1:

<table>
<thead>
<tr>
<th>$Z$</th>
<th>11</th>
<th>10</th>
<th>9</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R/R_0$</td>
<td>2.485</td>
<td>2.398</td>
<td>2.303</td>
<td>2.197</td>
<td>2.079</td>
<td>1.946</td>
<td>1.792</td>
<td>1.609</td>
<td>1.386</td>
<td>1.099</td>
<td>0.693</td>
<td>0.000</td>
</tr>
<tr>
<td>Increase</td>
<td>0.087</td>
<td>0.095</td>
<td>0.105</td>
<td>0.118</td>
<td>0.134</td>
<td>0.154</td>
<td>0.182</td>
<td>0.223</td>
<td>0.288</td>
<td>0.405</td>
<td>0.693</td>
<td></td>
</tr>
<tr>
<td>«Acceleration»</td>
<td>0.008</td>
<td>0.010</td>
<td>0.013</td>
<td>0.016</td>
<td>0.020</td>
<td>0.028</td>
<td>0.041</td>
<td>0.065</td>
<td>0.117</td>
<td>0.288</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

And in more details, from 2.2 to 0 with a step 0.2:

<table>
<thead>
<tr>
<th>$Z$</th>
<th>2.2</th>
<th>2</th>
<th>1.8</th>
<th>1.6</th>
<th>1.4</th>
<th>1.2</th>
<th>1</th>
<th>0.8</th>
<th>0.6</th>
<th>0.4</th>
<th>0.2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R/R_0$</td>
<td>1.163</td>
<td>1.099</td>
<td>1.030</td>
<td>0.956</td>
<td>0.875</td>
<td>0.788</td>
<td>0.693</td>
<td>0.588</td>
<td>0.470</td>
<td>0.336</td>
<td>0.182</td>
<td>0.000</td>
</tr>
<tr>
<td>Increase</td>
<td>0.065</td>
<td>0.069</td>
<td>0.074</td>
<td>0.080</td>
<td>0.087</td>
<td>0.095</td>
<td>0.105</td>
<td>0.118</td>
<td>0.134</td>
<td>0.154</td>
<td>0.182</td>
<td></td>
</tr>
<tr>
<td>«Acceleration»</td>
<td>0.004</td>
<td>0.005</td>
<td>0.006</td>
<td>0.007</td>
<td>0.008</td>
<td>0.010</td>
<td>0.013</td>
<td>0.016</td>
<td>0.020</td>
<td>0.028</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This property of magnification of distances from the past to the future for an even stride \( \Delta z \) in model of the Static Universe matches representation not only about expansion, but also about the sped up expansion of space in the models viewed by S.Perlmutter.[14] Thereby, all calculated changing properties related to a change of volume in models with expanded space: densities, concentrations, temperatures, a metallicity, etc., in model of the Static Universe have the right to be stationary values! Accordingly, the overdense and superhot state of substance, reionization and other properties of "Big Bang", as well as the moment of "a Universe birth» and well-known «the first three minutes» are absent here.

Generally, different cosmological theories have the right to treat differently the observational data gained by us is exclusive from an received electromagnetic radiation. The direct data in cosmology are absent. Therefore, in particular, statements about of all properties of a state of a matter and space during the first three minutes of the Universe should be not in an assertive form, but in subjunctive, with the binding preface before each deduction: «If to believe in the theory with expanding space, then...»

6. Conclusions
In article the deduction of the Basic cosmological formula (the formula (3), (6) is shown, (8)) is demonstrated on a minimum value of requirements in model of the Static Universe, the estimate of value of the usual mechanism of a variance of a velocity of light in intergalactic plasma is made, appearance of representation about the sped up dilation of space in the models viewed by S.Perlmutter is explained.

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